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Correlations Between Levels for Stellar-Scintillometer-Derived Profiles of C_n^2

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) A stellar scintillometer measures the variance of stellar intensity for a first magnitude or greater star. This variance can be converted, by the use of spatial filters, into a profile of C_n^2, the index of refraction structure constant, for 7 different altitudes or levels. The C_n^2 values are not entirely instrument independent because there will be some overlap in the weighting functions used to separate the spatial frequencies. It is usually assumed that readings from levels 1, 4, and 7 are instrument independent. GL scientists obtained 192 such profiles during a measurement program at Pennsylvania State University (PSU) over the period 30 April to 6 May 1986. When linear correlation coefficients are calculated between levels for C_n^2 and Log (C_n^2) of this data set, results obtained seem to contradict the above stated assumption. Much less correlation is observed when calculated on a nightly basis. In fact, some nightly coefficients are significantly negative, which could not be due to weighting function overlap. We have investigated other mechanisms which could lead to a high degree of correlation across the entire data set. During the measurement program, atmospheric conditions changed from

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evening to evening. For example, the jet stream passed through several times affecting the entire altitude range of the scintillometer. As a result of jet stream activity, C_n^2 is not a stationary variable which in turn suggests two approaches. In the first approach, we treat C_n^2 as a completely random variable on each evening with mean and standard deviation as actually measured for that evening. We can then generate the appropriate number of profiles for each evening to see how much correlation is produced when the nightly results are merged, giving 192 profiles. A substantial amount of correlation results. The second approach taken was to subtract the appropriate nightly means from each profile and recalculate the coefficients. This generally results in reduced correlations and in some cases, substantially reduced correlations between levels. We conclude that correlation studies do not indicate that, except for low lying adjacent levels, C_n^2 values obtained using the scintillometer are instrument dependent.

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PREFACE

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Correlations Between Levels for Stellar Scintillometer Derived Profiles of C_n^2

1. INTRODUCTION

In late April and early May 1986, Air Force Geophysics Laboratory (AFGL) personnel from the Atmospheric Optics Branch participated in a coordinated field program at Pennsylvania State University (PSU)[1]. The purpose of this program was, in part, to compare altitude profiles of C_n^2, the refractive index structure parameter[2], obtained from different measuring devices and to use the acquired data for checking the reliability of certain models for C_n^2. Profiles of this parameter can be obtained using a stellar scintillometer. The one used at PSU by AFGL is a modification of that originally developed by Ochs et al[3]. This instrument measures the variance of stellar intensity for a first magnitude or greater star. This variance, by use of spatial filters, can be converted into a spatially and temporally averaged profile of C_n^2 for 7 slant path levels where, by slant path level, we mean distance measured along the line of sight from the

(Received for publication 10 August 1990)

[1]Markson, R. M., Anderson, B. W., Fairall, C. W. , Thomson, D. W., White, A. B. and Syrett, W. J. (1989), *Atmospheric Turbulence Measurements in Support of Adaptive Optics Technology*, RADC-TR-89-289, ADA220445.

[2]Hufnagel, R. E. (1978), Propagation Through Atmospheric Turbulence, in *The Infrared Handbook*, USGPO, Washington D. C., Chap. 6, 1-56.

[3]Ochs, G. R., Wang, T., Lawrence, R. S. and Clifford, W. S. (1976), Refractive Turbulence Profiles Measured by One-Dimensional Spatial Filtering, *Appl. Opt.*, 15: 2504-2510.

instrument to the chosen star. Because the elevation angle for the star used is known as a function of time, the slant path distances can be converted to obtain the corresponding altitude above ground.

The C_n^2 values obtained for adjacent levels are not entirely instrument independent because there will be some overlap in the weighting functions used to separate the spatial frequencies. It is usually assumed that readings from levels 1, 4, and 7 are instrument independent. Figure 1 shows a typical scintillometer-derived profile for C_n^2. The scale shown has been chosen to show the usual altitude range of interest for this parameter and its typical variation. We see that the scintillometer does not provide information regarding C_n^2 for high and low altitudes but does give a profile from about 2 km to 17 km. It should be emphasized that the "points" on this graph really represent values centered at the indicated altitude but come from averaging over distances of the order of a kilometer and a time duration of 2-3 minutes.

It has been brought to our attention by one of the co-investigators in this measurement program from Rome Air Development Center (RADC)[4] that correlation coefficients for C_n^2 values derived from AFGL data (192 profiles) between different levels seem surprisingly large. Furthermore, this phenomenon has been observed for other scintillometer data sets from other measurement programs and the conclusion is that the assumption that the values of C_n^2 from levels 1, 4, and 7 are independent is incorrect. RADC also used a stellar scintillometer at PSU but their data (140 profiles) does not lead to such a high level of correlation.

In the absence of any other considerations, the correlation coefficients calculated by RADC for the AFGL scintillometer data would indicate that, for all levels, C_n^2 values are instrument-dependent, contradicting the above stated assumption of independence between levels 1, 4, and 7. It is the purpose of this report to further consider these observations and to suggest other possible reasons for the high degree of correlation observed.

2. CALCULATIONS

Data is available for the evenings of 30 April, 2 May, 3 May, 4 May, 5 May and 6 May. The number of profiles for each evening are 12, 39, 56, 26, 16, and 43 respectively, a total of 192.

[4]Stebbins, D. W. (1987), *Analysis of the Inter-layer Independence of Stellar Scintillometer Profiles of C_n^2*, RADC-TR-87-156, ADA189372.

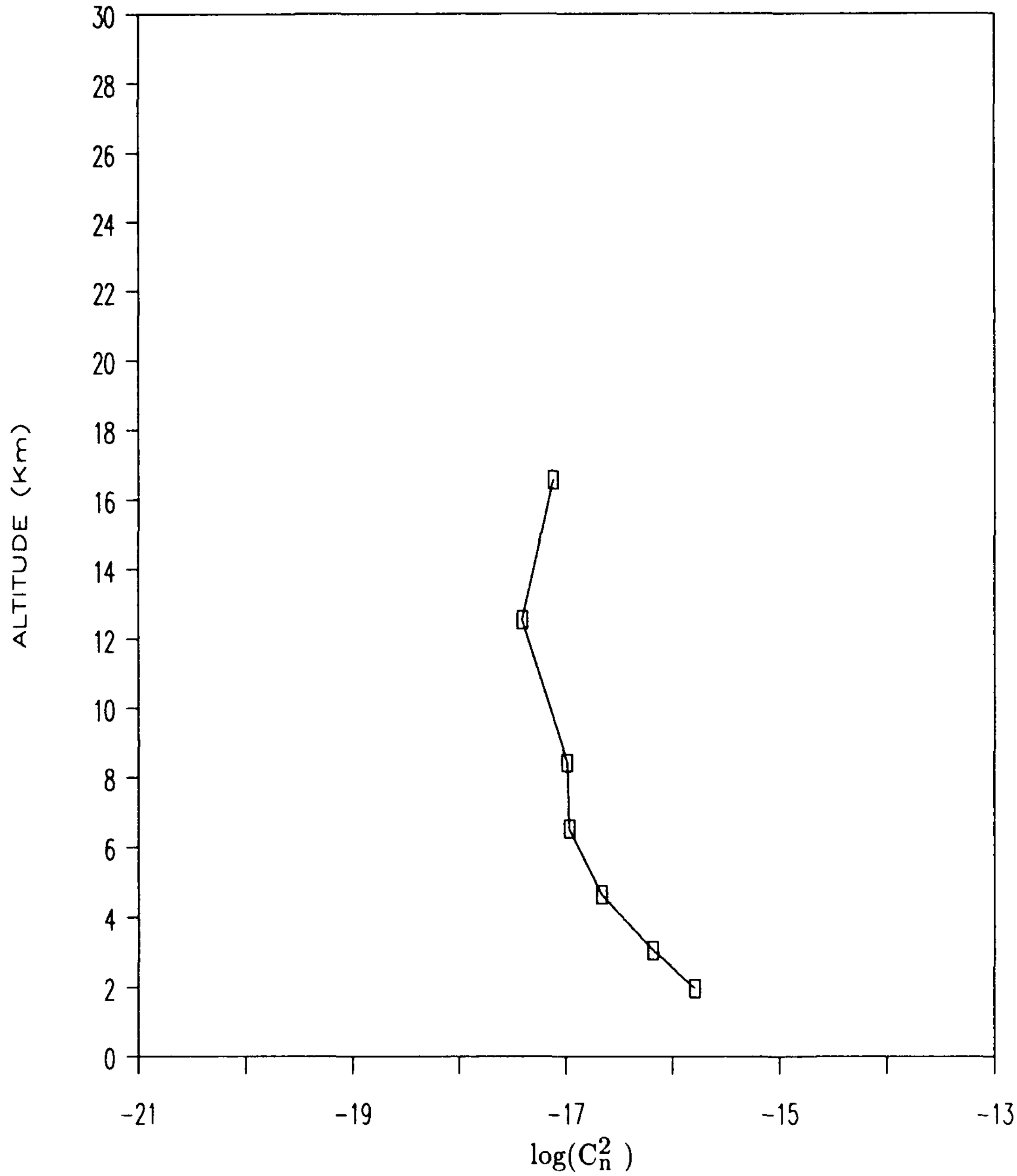

Figure 1. A typical profile obtained using a stellar scintillometer. The units of C_n^2 are $m^{-\frac{2}{3}}$.

2.1 Altitude versus Level Number

We began by looking at the difference between slant path levels and altitude above ground for the AFGL scintillometer profiles. During the PSU program, the instrument was pointed at the same star (Arcturus) at a near zenith position. The actual altitude for each measurement was calculated. Then the mean and standard deviation for all measurements were calculated. (See Table 1.) Because the standard deviation is of the order of 5 percent of the mean for each slant path level, we shall assume that the C_n^2 values from a given level correspond to the same altitude.

2.2 Correlation and Regression

The methods of linear regression and correlation[5] are frequently used to investigate the relationship between two (or more) variables. By the method of least squares the best straight line fit between two data sets can be obtained. The linear correlation coefficient, r, measures the strength and direction of the linear association of the two variables. The square of the correlation coefficient is the fraction of the variation of one variable that is explained by least squares regression on the other variable. It should be noted that correlations based on averages are usually too high when applied to individual values[6].

We have calculated correlation coefficients between each level for both C_n^2 and $\log(C_n^2)$ for each evening and for the entire program. These results are presented in Tables 2-8. Each table presents, in matrix form, two sets of correlation coefficients: $r_n(i,j)$ and $r_L(i,j)$ for the correlations of the numerical and logarithmic values of C_n^2 between level i and j. Tables 2-7 are for each evening and Table 8 contains results for the entire measurement program.

When we look at the results for all 192 profiles, we see for both C_n^2 and $\log(C_n^2)$ a significant positive correlation between all levels. Correlation coefficients decrease as the altitude separation between levels increases. Level 1 (at 2 km) and level 2 (at 3.1 km) are the closest and show the largest correlation. Levels 1 and 7 (at 16.6 km) show the least.

For the entire data set a picture much different than that of any individual

[5]See, for example, Brockett, P. and Levine, A. (1984), *Statistics and Probability and Their Applications*, chapter 9, Saunders College Publishing.

[6]For a detailed example of this phenomenon see Moore, D. and McCabe, G. (1989), *Introduction to the Practice of Statistics*, p. 210, W. H. Freeman.

Table 1. Mean altitude and standard deviation for each level.

Level	Mean Altitude (Km)	Standard Deviation (Km)
1	1.97	0.11
2	3.05	0.17
3	4.66	0.26
4	6.54	0.36
5	8.42	0.46
6	12.54	0.69
7	16.57	0.91

Table 2. Correlation coefficients between levels for C_n^2: 30 April 1986 (12 profiles).

	1	2	3	4	5	6	7
1		0.96	0.92	0.51	0.70	−0.04	0.44
2			0.97	0.36	0.61	0.05	0.38
3				0.43	0.62	0.26	0.39
4					0.86	0.11	0.67
5						−0.04	0.70
6							-0.01
7							

Correlation coefficients between levels for $\log(C_n^2)$: 30 April 1986 (12 profiles).

	1	2	3	4	5	6	7
1		0.96	0.95	0.72	0.70	0.18	0.67
2			0.99	0.57	0.63	0.15	0.54
3				0.61	0.66	0.22	0.50
4					0.86	0.09	0.58
5						−0.04	0.62
6							0.02
7							

Table 3. Correlation coefficients between levels for C_n^2: 2 May 1986 (39 profiles).

	1	2	3	4	5	6	7
1		0.86	0.43	-0.26	-0.25	0.17	0.08
2			0.71	-0.17	-0.18	0.26	0.12
3				0.43	0.28	0.34	0.06
4					0.88	0.05	0.14
5						-0.06	0.24
6							0.24
7							

Correlation coefficients between levels for $\log(C_n^2)$: 2 May 1986 (39 profiles).

	1	2	3	4	5	6	7
1		0.81	0.44	-0.31	-0.28	0.04	0.09
2			0.72	-0.31	-0.31	0.15	0.10
3				0.22	0.08	0.29	0.04
4					0.90	-0.12	0.05
5						-0.15	0.15
6							0.23
7							

Table 4. Correlation coefficients between levels for C_n^2: 3 May 1986 (56 profiles).

	1	2	3	4	5	6	7
1		0.92	0.85	0.39	0.30	0.24	0.27
2			0.93	0.42	0.25	0.46	0.38
3				0.65	0.42	0.31	0.34
4					0.87	0.10	0.36
5						0.12	0.34
6							0.60
7							

Correlation coefficients between levels for $\log(C_n^2)$: 3 May 1986 (56 profiles).

	1	2	3	4	5	6	7
1		0.89	0.70	0.16	0.20	0.12	0.28
2			0.90	0.35	0.35	0.24	0.33
3				0.62	0.53	0.24	0.49
4					0.83	0.19	0.43
5						0.22	0.34
6							0.47
7							

Table 5. Correlation coefficients between levels for C_n^2: 4 May 1986 (24 profiles).

	1	2	3	4	5	6	7
1		0.33	-0.03	-0.07	0.20	0.29	0.18
2			0.73	0.34	0.06	-0.14	-0.55
3				0.65	0.11	-0.24	-0.54
4					0.41	-0.03	-0.15
5						0.35	0.17
6							0.37
7							

Correlation coefficients between levels for $\log(C_n^2)$: 4 May 1986 (24 profiles).

	1	2	3	4	5	6	7
1		0.19	-0.06	0.01	0.21	0.17	-0.05
2			0.57	0.39	-0.04	-0.18	-0.49
3				0.81	0.21	-0.19	-0.44
4					0.45	0.05	-0.13
5						0.23	-0.03
6							0.17
7							

Table 6. Correlation coefficients between levels for C_n^2: 5 May 1986 (16 profiles).

	1	2	3	4	5	6	7
1		0.53	0.40	0.14	-0.23	0.25	-0.12
2			0.95	0.30	-0.13	0.20	-0.13
3				0.40	-0.03	0.15	-0.08
4					0.53	0.04	0.10
5						-0.12	0.45
6							0.03
7							

Correlation coefficients between levels for $\log(C_n^2)$: 5 May 1986 (16 profiles).

	1	2	3	4	5	6	7
1		0.54	0.39	0.26	-0.37	0.24	-0.17
2			0.89	0.13	-0.16	0.17	-0.11
3				0.10	-0.14	0.12	0.03
4					0.06	-0.01	0.06
5						-0.11	0.45
6							0.03
7							

Table 7. Correlation coefficients between levels for C_n^2: 6 May 1986 (43 profiles).

	1	2	3	4	5	6	7
1		0.88	0.85	0.80	0.83	0.50	0.25
2			0.99	0.86	0.64	0.28	0.27
3				0.89	0.62	0.26	0.28
4					0.71	0.40	0.27
5						0.63	0.12
6							0.04
7							

Correlation coefficients between levels for $\log(C_n^2)$: 6 May 1986 (43 profiles).

	1	2	3	4	5	6	7
1		0.77	0.72	0.52	0.66	0.66	0.17
2			0.88	0.73	0.62	0.43	0.21
3				0.88	0.68	0.38	0.20
4					0.69	0.36	0.13
5						0.57	0.04
6							0.08
7							

Table 8. Correlation coefficients between levels for C_n^2: 30 April-6 May 1986 (192 profiles).

	1	2	3	4	5	6	7
1		0.91	0.82	0.44	0.42	0.40	0.37
2			0.93	0.52	0.45	0.52	0.43
3				0.72	0.57	0.44	0.40
4					0.87	0.34	0.46
5						0.42	0.56
6							0.60
7							

Correlation coefficients between levels for $\log(C_n^2)$: 30 April-6 May 1986 (192 profiles).

	1	2	3	4	5	6	7
1		0.88	0.76	0.54	0.61	0.62	0.52
2			0.92	0.66	0.64	0.53	0.43
3				0.77	0.68	0.49	0.41
4					0.75	0.42	0.38
5						0.54	0.49
6							0.55
7							

evening emerges except for low lying adjacent layers. Consider, for example, $r_n(1,7)$ which for successive evenings takes on the values 0.44 (12 profiles), 0.08 (39 profiles), 0.27 (56 profiles), 0.18 (26 profiles), −0.12 (16 profiles) and 0.25 (43 profiles), yielding a weighted mean of 0.19. For all 192 profiles $r_n(1,7) = 0.37$, that is, about a factor of two larger than nightly measurements would suggest. Similarly for $r_n(6,7)$ we have on a nightly basis −0.01, 0.24, 0.60, 0.47, 0.03, and 0.04, giving a weighted mean of 0.30, whereas for the entire sample we have $r_n(6,7) = 0.60$.

Statistically significant negative correlations have been obtained on some evenings. For example, for 2 May, based on 39 profiles, $r_L(1,4) = -0.31$ which yields a t-statistic[7] of −1.98, which is significant at the 0.05 level. For 4 May based on 26 profiles we find that $r_L(2,7) = -0.49$ which yields a t-statistic of −2.75 which is significant at the 0.01 level.

We have also have calculated for C_n^2 and for $\log(C_n^2)$ correlation coefficients with altitude for the entire data set, obtaining values of −0.31 for C_n^2 and −0.50 for $\log(C_n^2)$.

For each evening and for the entire sample we have calculated the slope and intercept of the least squares line for C_n^2 and $\log(C_n^2)$ with altitude.

2.3 Scatter Plots

Scatter plots have been produced to illustrate the above discussion. The ones presented here are all for $\log(C_n^2)$ rather than for C_n^2 because including even one outlier of C_n^2 can lead to a rather distorted plot. Figure 2 shows the graph of $\log(C_n^2)$ for levels 1 versus level 7 for the entire data set. This plot is typical for a moderate amount of correlation. We also show the least squares line. Figure 3 shows a scatter plot for $\log(C_n^2)$ for level 1 versus level 2 and shows a high degree of correlation. In Figure 4 we display a typical scatter diagram for a negative correlation coefficient and in Figure 5 such a plot for a near zero correlation.

3. POSSIBLE CAUSES OF HIGH CORRELATION

Various mechanisms exist, other than the scintillometer itself, that could lead to a high degree of correlation of C_n^2 and $\log(C_n^2)$ values between levels. During the course of the PSU measurement program, atmospheric conditions were not steady from evening to evening, nor during the course of a single evening. Jet stream passages were

[7] Brockett, P. and Levine, A. (1984), *Statistics and Probability and Their Applications*, p. 320, Saunders College Publishing, 1984.

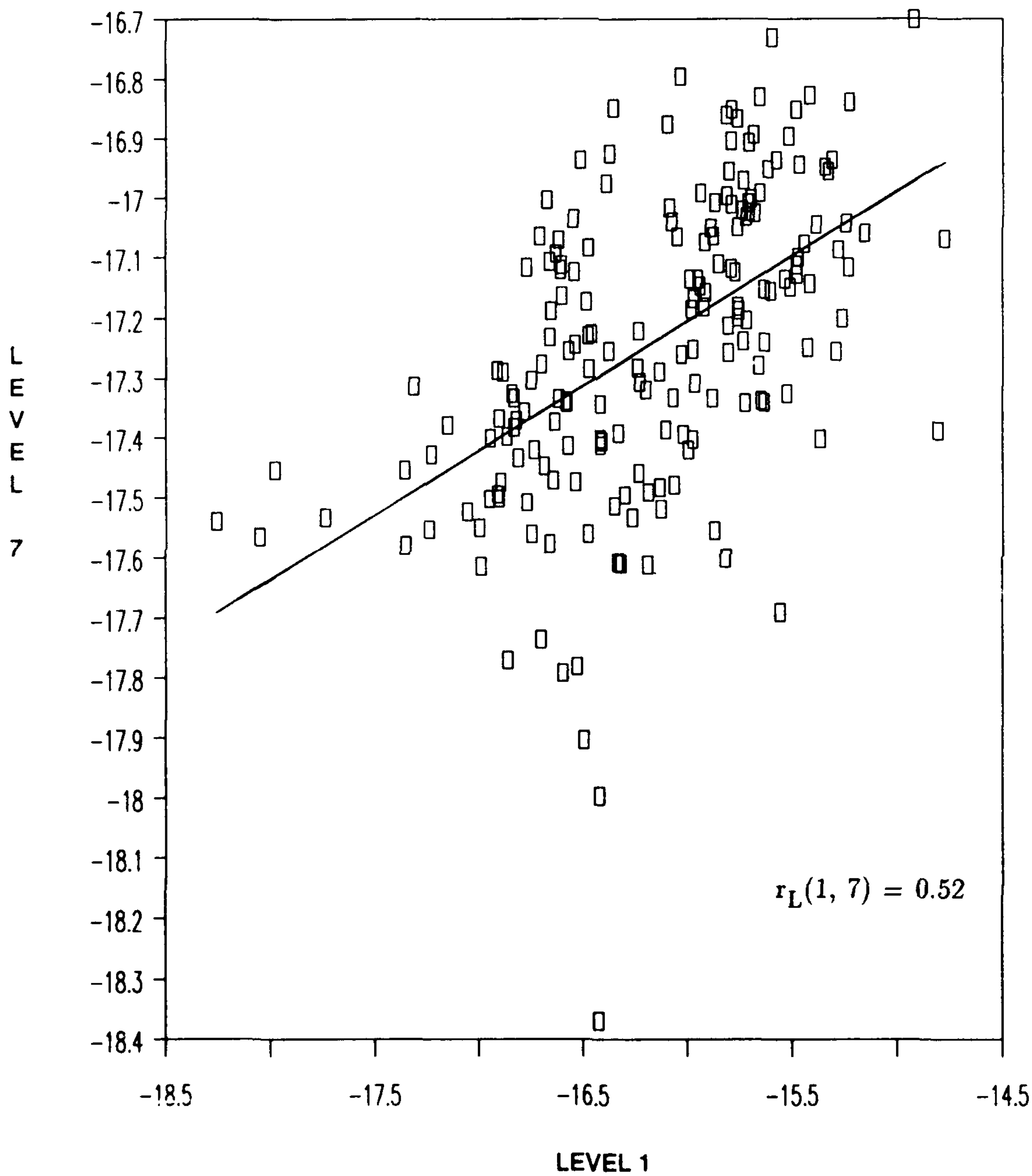

Figure 2. $\log(C_n^2)$: Level 1 versus Level 7; 30 April-6 May 1986. 192 profiles. Shown for comparison is the least squares line.

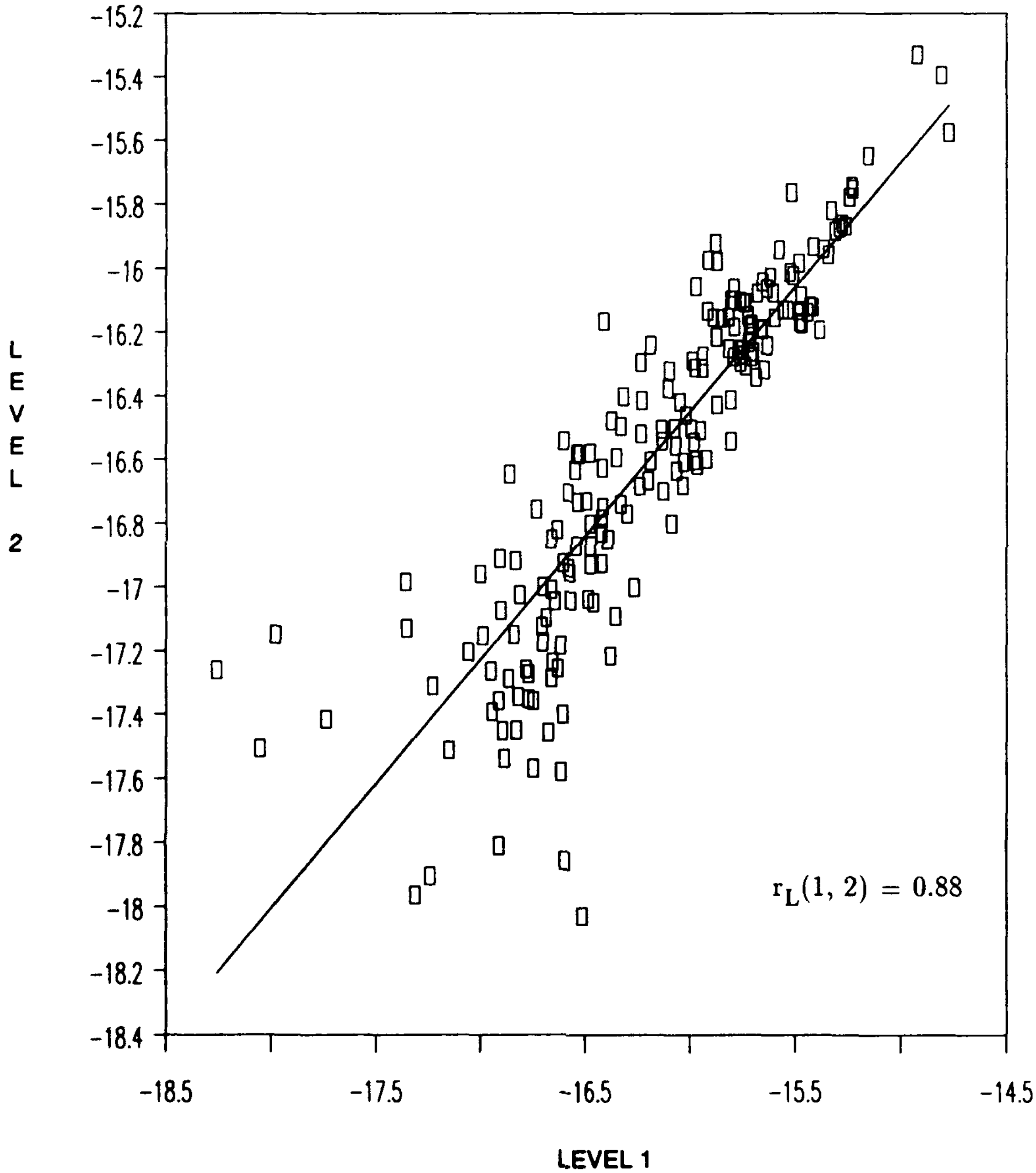

Figure 3. $\log(C_n^2)$: Level 1 versus Level 2; 30 April-6 May 1986. 192 profiles. Shown for comparison is the least squares line.

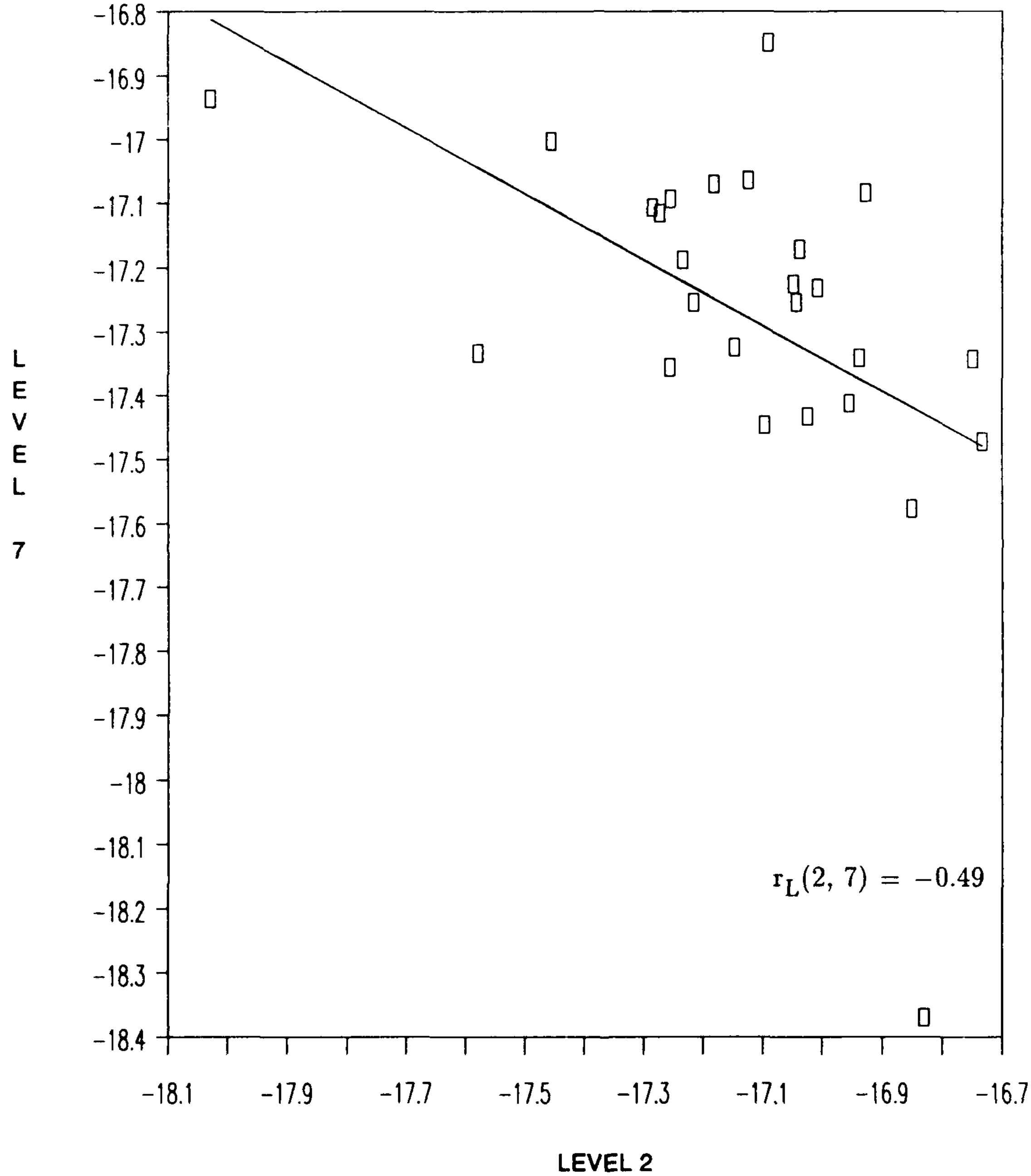

Figure 4. $\log(C_n^2)$: Level 2 versus Level 7; 4 May 1986.
26 profiles. Shown for comparison is the least squares line.

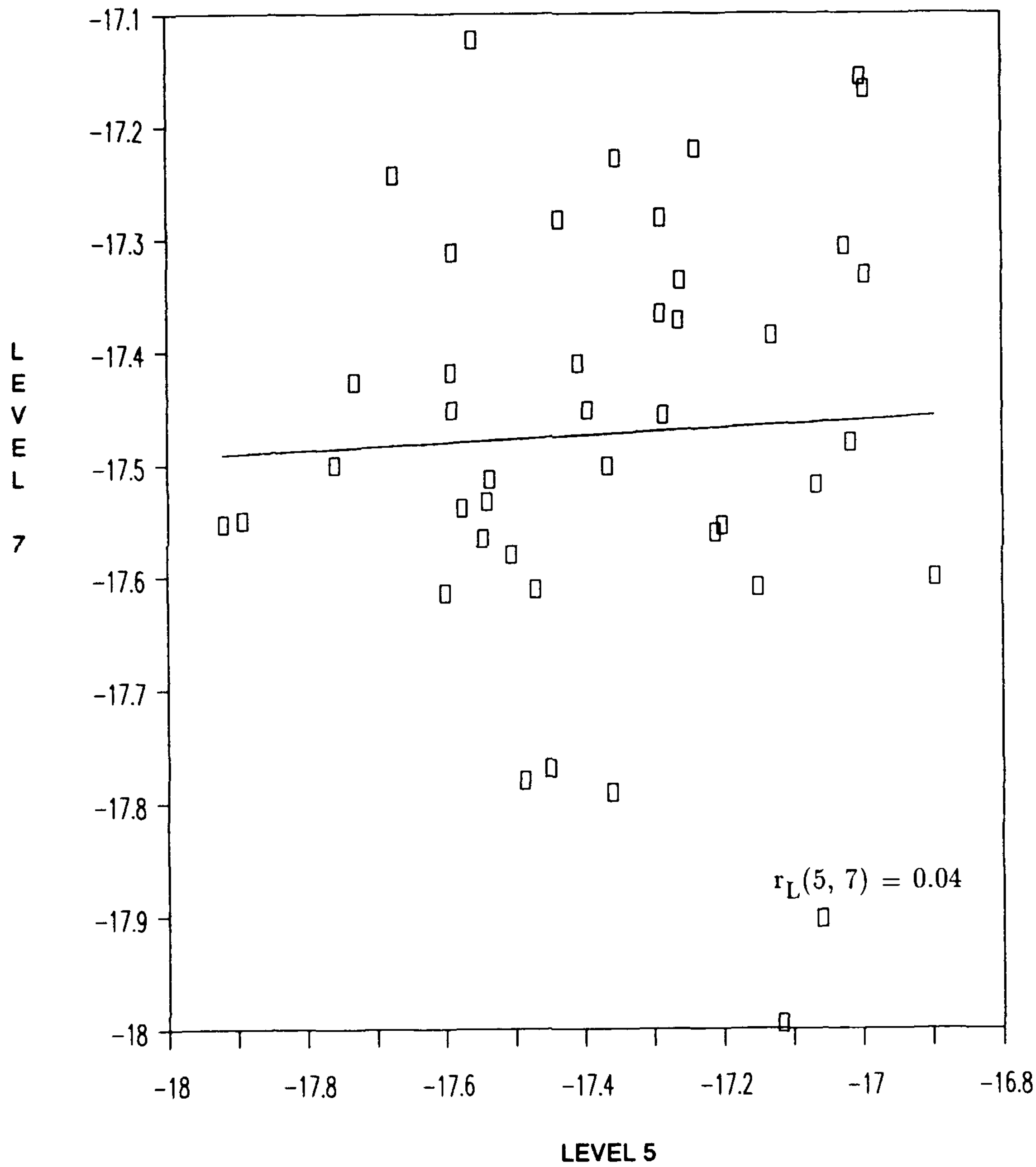

Figure 5. $\log(C_n^2)$: Level 5 versus Level 7; 5 May 1986.
43 profiles. Shown for comparison is the least squares line.

observed over this time period resulting in significant changes in the various turbulence parameters measured[8]. Jet stream effects, for example, could influence atmospheric conditions across most of the scintillometer measurement range. After looking at a simple example in Section 3.2 we estimate the possible effects of changing atmospheric conditions in two ways in Section 3.3.

If instantaneous localized measurements of C_n^2 were available at different altitudes we would expect there to be no correlation. However, scintillometer profiles involve a good deal of spatial and temporal averaging. This effect will be addressed in Section 3.4.

3.1 Stationarity and Independence

A stationary variable[9] is one which is distributed about some definite mean with a definite standard deviation and these values do not change with time. Two variables are said to be independent if changes in one do not influence the other. To illustrate the effects of stationarity and independence (or the lack thereof) we construct the following simple data sets.

Let X(i, j) be a random variable uniformly distributed over the interval [i, j]. X(i, j) is clearly a stationary variable with mean $\frac{i+j}{2}$ and standard deviation $\frac{1}{2\sqrt{3}}(j - i)$. In Table 9-A, in column one are shown 12 values of X(1, 3) and in column two are shown 12 values of X(2, 4). Below the table is shown the correlation coefficient for these two independent sets. An individual value of this correlation coefficient is inconclusive. We generated a hundred such tables and calculated the corresponding correlation coefficients and found an average correlation coefficient of −0.01 (with a standard deviation of 0.33) which is what we would expect: the correlation between two independent stationary variables should be zero. Tables 9-B and 9-C are self-explanatory.

Consider Table 10. It is formed by merging Tables 9-A, B and c. A high degree of correlation is exhibited. This is not a coincidence: when 100 such tables were generated, we found that the average correlation coefficient was 0.89 with a standard deviation of 0.02. We can explain this correlation in at least two ways:

[8]Beland, R. (1990), Geophysics Laboratory, private communication.

[9]Cox, D. R. and Lewis, P. S. (1960), *The Statistical Analysis of Series of Events*, p. 60, John Wiley & Sons.

Table 9. Two sets of random numbers and their correlations, r.

A. The first set is drawn from the interval (1,3) and the second from (2,4)

B. The first set is drawn from the interval (3,5) and the second from (4,6)

C. The first set is drawn from the interval (5,7) and the second from (6,8)

A		B		C	
X(1,3)	X(2,4)	X(3,5)	X(4,6)	X(5,7)	X(6,8)
1.84	2.88	4.76	4.95	5.92	7.93
1.32	2.56	3.10	5.38	5.62	7.95
1.94	2.31	3.93	4.08	6.96	6.64
2.01	3.49	3.57	4.48	5.90	6.83
2.36	3.98	3.77	4.15	6.13	7.78
1.49	3.74	3.20	4.28	6.16	6.55
1.46	2.34	3.93	4.41	5.33	6.48
1.61	2.83	3.94	5.58	6.71	6.72
2.11	2.99	3.34	4.19	6.89	7.56
2.85	2.06	4.59	4.63	5.51	7.58
2.85	2.66	3.84	5.83	6.41	7.66
2.83	2.66	3.97	5.11	5.92	6.34
r =	-0.14	r =	0.11	r =	-0.08

(i) The two columns are not independent; although each group of twelve pairs were generated independently, these three groups were not merged at random.

(ii) If we think of each column as a time series with measurements made every second, then every 12 seconds the mean of each set changes.

3.2 Possible Effects on $r_L(i, j)$[10]

We think that a good deal of the observed correlation in Table 8, especially for non-adjacent levels, can be explained in terms of the above example. Atmospheric conditions varied from one night to the next during the measurement program. Nightly means and standard deviations for C_n^2 and $\log(C_n^2)$ vary considerably. See Table 11 for nightly values for $\log(C_n^2)$. The fact that these nightly values do differ suggests that C_n^2 is not a stationary variable and that lumping all 192 profiles into one data set and then calculating correlation coefficients between levels is not unlike merging Tables 9-A,B and C to form Table 10. In order to see the type of correlation that may be due to this phenomenon we take two approaches.

[10]Because $r_n(i, j)$ and $r_L(i, j)$ are comparable in magnitude, we concentrate on the latter.

Table 10.
The result of merging
Tables 9A, B and C

Z1	Z2
1.84	2.88
1.32	2.56
1.94	2.31
2.01	3.49
2.36	3.98
1.49	3.74
1.46	2.34
1.61	2.83
2.11	2.99
2.85	2.06
2.85	2.66
2.83	2.66
4.76	4.95
3.10	5.38
3.93	4.08
3.57	4.48
3.77	4.15
3.20	4.28
3.93	4.41
3.94	5.58
3.34	4.19
4.59	4.63
3.84	5.83
3.97	5.11
5.92	7.93
5.62	7.95
6.96	6.64
5.90	6.83
6.13	7.78
6.16	6.55
5.33	6.48
6.71	6.72
6.89	7.56
5.51	7.58
6.41	7.66
5.92	6.34

r = 0.90

3.2.1 Treat C_n^2 as a "Nightly" Random Variable

Suppose we assume that on any given night C_n^2 has a uniform distribution whose mean and standard deviation at each level are as given in Table 11. This assumption will underestimate the amount of correlation from level to level in that C_n^2 is most likely log normally distributed[11]. Furthermore, during the course of an evening, atmospheric conditions may have changed. We now take the nightly mean and standard deviation for $\log(C_n^2)$ and generate n random variables (where n is the number of profiles for the night in question) centered on this mean in a range of width $2\sqrt{3}$ times the nightly standard deviation[12]. We then combine results for each together as in the above example to get 192 profiles. We then calculate correlation coefficients between levels using this data. We then repeated this procedure 100 times. Table 12 shows the resulting average correlation coefficients. For non-adjacent levels especially, the resulting correlations are comparable to those of Table 8.

Table 11. Nightly means and standard deviations for $\log(C_n^2)$ by level. The units of C_n^2 are $m^{-\frac{2}{3}}$ and n is the number of profiles.

DATE	n	1	2	3	4	5	6	7
4/30	12	−16.123 0.538	−16.564 0.486	−16.749 0.316	−16.935 0.180	−16.948 0.192	−17.187 0.191	−17.117 0.196
5/2	39	−15.667 0.202	−16.107 0.156	−16.512 0.135	−16.944 0.291	−16.934 0.197	−17.050 0.163	−17.023 0.133
5/3	56	−15.800 0.426	−16.291 0.334	−16.709 0.306	−17.117 0.339	−17.077 0.232	−17.138 0.223	−17.198 0.197
5/4	26	−16.599 0.130	−17.130 0.266	−17.548 0.331	−17.639 0.412	−17.353 0.191	−17.279 0.227	−17.271 0.279
5/5	16	−16.790 0.184	−17.350 0.221	−17.853 0.446	−17.811 0.458	−17.766 0.407	−17.566 0.146	−17.421 0.134
5/6	43	−16.661 0.601	−16.796 0.488	−17.065 0.427	−17.400 0.505	−17.368 0.252	−17.500 0.212	−17.473 0.191

[11] Loos, G. and Hogge, C. (1979), Turbulence of the Upper Atmosphere and Isoplanatism, *Appl. Opt.*, 18: 2654-2661.

[12] One can show that a uniform distribution centered on x with width $2\sqrt{3}$ s will have a standard deviation of s.

3.2.2 Recalculate Table 8 with Nightly Means Subtracted

A second approach in dealing with the non-stationary nature of C_n^2 is to subtract out the nightly means before calculating correlation coefficients. The result of doing this is shown in Table 13. This table should again be compared to Table 8. Correlation has been significantly reduced. For example, $r_L(1, 7)$ has dropped from 0.52 down to 0.20. The amount of variation in $\log(C_n^2)$ at level 7 that can be inferred from variation at level 1 has dropped from 27 percent down to $r_L^2(1, 7) = 4$ percent. We further note that $r_L^2(1, 4) = 8$ percent and $r_L^2(4, 7) = 2$ percent which would indicate only a slight interdependence between levels 1, 4, and 7. This residual correlation may be due to the scintillometer but could also be due to non-stationarity of C_n^2 on a given evening.

3.3 Altitude Dependence of C_n^2

Various models for C_n^2 have been proposed. In most of these altitude is directly or indirectly the dominant parameter. Any model of the form $C_n^2(z) = C_n^2(z_o)\cdot f(z)$ where $f(z)$ is any function of altitude and $C_n^2(z_o)$ is some reference value would predict a high degree of correlation, but not necessarily linear correlation. Some such models include that of Kaimal as modified by Walters and Kunkel,[13] that of Battles, Murphy and Noonan[14] and that of Kukharets and Tsang,[15] each of which would yield a high correlation of C_n^2 from level to level. We are not saying that these models do apply to the altitude range covered by this data but are suggesting that the high degree of correlation observed may be due to the fact that, although very different physical mechanisms are producing turbulence at the different levels sampled, when sufficient data are used we expect that on the whole C_n^2 will be a function of altitude. This would explain the difference between nightly results versus that observed over the entire measurement program. From night to night differing atmospheric conditions exist. When these are averaged over all six nights C_n^2 becomes correlated due to the fact that it is overall a decreasing function of altitude.

The correlation coefficients calculated for C_n^2 and $\log(C_n^2)$ with altitude are a further indication that altitude, not instrumentation, is affecting the values of $r_n(i,j)$

[13]Walters, D. L. and Kunkel, K. E. (1981), Atmospheric Transfer Function for Desert and Mountain Locations: The Atmospheric Effects on r_0, *J. Opt. Soc. Am.*, 71: 397-405.

[14]Battles, F. P., Murphy, E. A. and Noonan, J. P. (1988), The Contribution of Atmospheric Density to the Drop-off Rate of C_n^2, *Physica Scripta*, 37: 151-153.

[15]Kukharets, V. P. and Tsang, L. R. (1980), Structure Parameter of the Refractive Index in the Atmospheric Boundary Layer, *IZV Acad Sci USSR, Atmos. Oceanic Phys.*, 16: 397-405.

Table 12. Correlation coefficients obtained by assuming that $\log(C_n^2)$ is randomly distributed with a mean and standard deviation as given in Table 11. These results are based on the average of 100 simulations and each has a standard deviation of about 0.06.

	1	2	3	4	5	6	7
1		0.45	0.50	0.39	0.47	0.46	0.41
2			0.50	0.37	0.43	0.37	0.26
3				0.44	0.48	0.40	0.34
4					0.40	0.33	0.29
5						0.46	0.38
6							0.39
7							

Table 13. Correlation coefficients for $\log(C_n^2)$ obtained from the scintillometer data after the nightly means given in Table 11 have been subtracted out.

	1	2	3	4	5	6	7
1		0.79	0.62	0.28	0.29	0.30	0.20
2			0.83	0.44	0.30	0.23	0.14
3				0.64	0.38	0.20	0.14
4					0.61	0.16	0.15
5						0.20	0.19
6							0.24
7							

and $r_L(i,j)$. Should a model of the above suggested form hold we would expect that a least squares line for $\log(C_n^2)$ from level to level would have a slope of near unity. Our calculations for adjacent levels yield slopes of order 0.8.

If the observed correlations were primarily instrument caused, we would expect a similar pattern on any given evening similar to that exhibited overall: statistically significant positive correlation that decreases with level separation. Weighting function overlap can only result in positive correlation, yet significant negative correlation has been observed on several occasions.

4. CONCLUSIONS

Because $r_L^2(i, j)$ can be interpreted as giving the percent variation in $\log(C_n^2)$ at level i that is explained by least squares regression on $\log(C_n^2)$ at level j, we present Tables 14-16 which give the values of $r_L^2(i, j)$ for the 192 profiles as measured (the square of Table 8), the values obtained by assuming that $\log(C_n^2)$ is uniformly distributed about the nightly mean (the square of Table 12) and the values of $r_L^2(i, j)$ obtained by subtracting out the nightly means (the square of Table 13.)

Suppose that we somewhat arbitrarily define "substantial correlation" to correspond to $r_L^2(i, j) \geq 0.20$ and "insubstantial correlation" to correspond to $r_L^2(i, j) \leq 0.10$. Then in Table 14, we see that there is substantial correlation between practically all pairs of levels. However, when we consider Table 16 we see that correlation between almost all non-adjacent levels is insubstantial, the exceptions being $r_L^2(1, 3) = 0.38$, $r_L^2(2, 4) = 0.19$ and $r_L^2(3, 5) = 0.14$. There is insubstantial correlation shown for all of the $r_L^2(i, j)$ for $i < j + 1$ when the nightly means have been removed.

In Table 15 we see that the non-stationarity of C_n^2 may have introduced substantial correlation between some levels, especially when it is remembered that this table was generated using assumptions that underestimate the values of $r_L(i, j)$ as noted in Section 3.2.1.

We conclude that values obtained for C_n^2 at levels 1, 4, and 7 are relatively independent of one another. This also appears to be true for all but low lying adjacent or next-adjacent levels.

Table 14. The square of the correlation coefficients between levels for $\log(C_n^2)$. April 30-May 6, 1986 (192 profiles). Compare with Table 8.

	1	2	3	4	5	6	7
1		0.77	0.58	0.29	0.37	0.38	0.27
2			0.85	0.44	0.41	0.28	0.18
3				0.59	0.46	0.24	0.17
4					0.56	0.18	0.14
5						0.29	0.24
6							0.30
7							

Table 15. Square of the correlation coefficients obtained by assuming that $\log(C_n^2)$ is randomly distributed with a mean and standard deviation as given by Table 11. These results are based on the average of 100 simulations. Compare with Table 12.

	1	2	3	4	5	6	7
1		0.20	0.25	0.15	0.22	0.21	0.17
2			0.25	0.14	0.18	0.14	0.07
3				0.19	0.23	0.16	0.12
4					0.16	0.11	0.08
5						0.21	0.14
6							0.15
7							

Table 16. Square of the correlation coefficients for $\log(C_n^2)$ obtained from the scintillometer data after the nightly means given in Table 11 have been subtracted out. Compare with Table 13.

	1	2	3	4	5	6	7
1		0.62	0.38	0.08	0.08	0.09	0.04
2			0.69	0.19	0.09	0.05	0.02
3				0.41	0.14	0.04	0.02
4					0.37	0.03	0.02
5						0.04	0.04
6							0.06
7							

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5. See, for example, Brockett, P. and Levine. A. (1984), *Statistics and Probability and Their Applications*, chapter 9, Saunders College Publishing.

6. For a detailed example of this phenomenon see Moore, D. and McCabe, G. (1989), *Introduction to the Practice of Statistics*, p. 210, W. H. Freeman.

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12. One can show that a uniform distribution centered on x with width 2 $\sqrt{3}$ s will have a standard deviation of s.

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15. Kukharets, V. P. and Tsang, L. R. (1980), Structure Parameter of the Refractive Index in the Atmospheric Boundary Layer, *IZV Acad Sci USSR Atmos. Oceanic Phys.*, 16: 397-405.